Rapid Measurement of Dielectric Constant and Loss Tangent

D. M. BOWIE AND K. S. KELLEHER†

Summary—**The problem of evaluating dielectric constant and loss tangent by the short-circuited-waveguide technique has been encountered continually in recent years in the study of artificial dielectric media and radome materials. In general, practical measurements have involved materials with low loss and dielectric constants less than 10. The analytical method normally applied to data on such materials requires laborious computation. The available graphical methods have not completely eliminated computation and have provided answers of unsatisfactory accuracy.**

The present paper describes rapid graphical techniques for evaluating dielectric constant and loss tangent from the quantities normally measured with the slotted line, using samples of arbitrarily chosen length. It begins with equations previously derived for the case of low-loss media. By use of a new parameter, the relationship between dielectric constant and the measured shift in standing-wave minimum is plotted in such a way that all possible values of dielectric constant within any predetermined range are read directly from the graph with no computation whatsoever. A graph can be readily prepared to apply over a full range of frequency to all sizes of rectangular waveguide.

With the dielectric constant known, a simplification in determining the loss tangent is possible, using half-wavelength samples. The loss tangent is obtained by direct recourse to a graph of loss tangent as a function of the standing wave ratio.

Dielectric Constant Measurement

Introduction

THE SHORTED-LINE method[1–4] for determining dielectric constant now utilized in many laboratories for evaluation of dielectric materials used in radomes, lenses, and other microwave components necessitates lengthy computational procedures after each experimental measurement. However, there is a method, introduced here, which reduces the time and eliminates all computation by using a graph for evaluation of the dielectric constant. This graph does not include the effect of loss in the dielectric, which has been treated at length,[4] but returns to the more common problem of dielectric materials of low loss. It has greatest similarity to graphs published by Redheffer,[3] but offers several advantages over that presentation.

In previous graphical methods, the ambiguity introduced by the cyclic nature of the tangent function, rather than being clearly expressed in a cyclical line set, was required to be introduced by repeatedly adjusting a parameter-value (usually one of length), each adjustment yielding a possible value of K_e. This has usually resulted in obscuring the relation between sample length and the order of ambiguity present. A second limitation is that the plotting range has sometimes been directly related to the ratio of sample length to wavelength, which for a given wavelength places an upper limit on length of sample which can be accommodated by lines within a given rectangular plotting region. The present method, without these inherent limitations, is adaptable to any selection of sample lengths. This is a useful feature in testing samples of artificial dielectrics, where long samples are needed in order that a sample contain a sufficient number of *inclusions*, on which the effective dielectric constant depends.

In the determination of dielectric constant, it is desirable to utilize a single graph showing the relationship between the dielectric constant and a measured quantity. Moreover, the dielectric constant should be read from the graph to an accuracy of better than 1 per cent, preferably to an accuracy of 0.5 per cent. A final requirement, peculiar to this particular problem, is that ambiguity in the dielectric constant value should be evident from the graph alone, and not require recomputation. It is felt that the graph described in this paper adequately satisfies all of these conditions.

Restrictions on the use of the graph are identical to those imposed in the method mentioned[3] in that the graph holds only for a fixed value of the parameter p, where p equals $(\lambda/\lambda_c)^2$, and holds for sample length of discrete values, arbitrarily selected. Subject to these conditions, the graph yields, to the desired degree of accuracy, all possible solutions for the dielectric constant within any preset limits. The philosophy behind this graph will be thoroughly discussed in order that it can be reproduced by the reader or extended to greater values of dielectric constant than had been encountered in the authors' work in the derivation of the graph.

Since we are considering only low-loss material, we begin with the two expressions given by Dakin and Works:[2]

$$\frac{-\lambda_g \tan 2\pi(x_0/\lambda_g)}{2\pi d} = \frac{\tan \beta_2 d}{\beta_2 d} \tag{1a}$$

$$K_e = \frac{1/\lambda_c^2 + (\beta_2 d)^2/(2\pi d)^2}{1/\lambda_c^2 + 1/\lambda_g^2} \tag{2a}$$

where λ_g and λ_c are guide wavelength and cutoff wavelength, respectively, β_2 the propagation constant in the dielectric sample, d the length of the sample, and x_0 the measured quantity shown in Fig. 1.

† Melpar, Inc., Falls Church, Va.

[1] S. Roberts and A. von Hippel, "A new method for measuring dielectric constant and loss in the range of centimeter waves," *J. Appl. Phys.*, vol. 17, pp. 610–619; June, 1946.

[2] T. W. Dakin and C. N. Works, "Microwave dielectric measurements," *J. Appl. Phys.*, vol. 18, pp. 789–796; September, 1947.

[3] R. M. Redheffer, "Technique of Microwave Measurements," McGraw-Hill Book Co., Inc., New York, N. Y., p. 625; 1947.

[4] R. M. Redheffer, R. C. Wildman, and W. O'Gorman, "The computation of dielectric constants," *J. Appl. Phys.*, vol. 23, pp. 505–508; May, 1952.

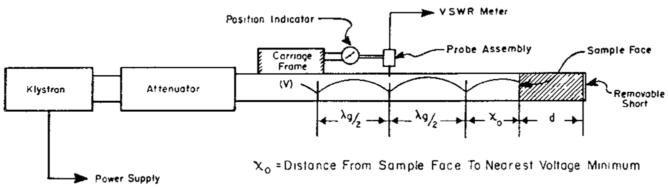

Fig. 1—Diagram of test apparatus with measured distances shown.

Analysis of Equations—Selection of Parameters

In order to clarify the above equations, in preparation for their expression in graphical form, two substitutions are made. The constant β_2 is replaced by its definition: $\beta_2 = 2\pi/\lambda_{gs}$, where λ_{gs} is the wavelength within the sample in the guide, and the parameter $p = (\lambda/\lambda_c)^2$ is introduced. In this way these equations are obtained in the following form:

$$\frac{\lambda_{gs}}{\lambda_g} = \frac{-\tan 2\pi(x_0/\lambda_g)}{\tan 2\pi(d/\lambda_{gs})} \tag{1b}$$

$$K_e = (1 - p)(\lambda_g/\lambda_{gs})^2 + p. \tag{2b}$$

Eq. (1b) clearly expresses the phase relationship across the air-sample interface in terms of existent wavelengths. The value of x_0 is limited by its definition to the range: $0 < x_0 < \lambda_g/2$; the value of d is limited to the maximum length of sample which the guide will accommodate; the ratio λ_{gs}/λ_g has values in general less than 1.

It has been arbitrarily decided that the final graph will use fixed values of the parameter $p = (\lambda/\lambda_c)^2$ and sample length d. For these fixed values, it is possible to solve the first equation for the measured quantity x_0 in terms of a new parameter: $q = \lambda_g/\lambda_{gs}$. Since p is a constant, the second equation gives the dielectric constant K_e directly in terms of the parameter q. Eliminating this parameter between the two expressions produces the desired relationship between the dielectric constant and the measured quantity.

The choice of sample length, d, can be used to minimize the difficulty caused by ambiguity inherent in rf dielectric constant measurements. If the approximate working range of K_e is known, a value of d can be found which will produce a single cycle of values of x_0 vs q; larger values of length will produce more than a single cycle, in which case a single value of x_0 corresponds to more than one value of the parameter q. The approximate limit of sample length, d', which presents no ambiguity between given maximum and minimum values of dielectric constant, is given by the following formula:

$$d' \leqq 0.6\lambda_g/\sqrt{K_e(\max) - K_e(\min)}.$$

In cases where estimates of the dielectric constant are not feasible, it is necessary to make two measurements using different sample lengths and to use the value of K_e which occurs in agreement for the two lengths. With this procedure, no prior knowledge of dielectric constant is needed. For convenience in testing, sample lengths can be made to be integral multiples of the narrow dimension of the test waveguide.

Method of Construction of Graphs

The procedure for obtaining K_e consists, first, in graphically recording values of x_0 vs assumed values of the parameter $q = \lambda_g/\lambda_{gs}$, for preselected values of d and λ_g expressed as their ratio. The choice of p, of course, determines λ_g. This preselection determines values of both the arguments, x_0/λ_g and d/λ_{gs}, in (1b). A sufficient number of q values are assumed to make feasible a graph of x_0 vs q over a preselected working range of the latter parameter.

The relationship between the dielectric constant K_e and the parameter q, expressed in (2), can be utilized in tabular form, together with the graph of (1), to yield x_0 vs K_e values for plotting. However, it has been found convenient to graph the function $K_e(q)$ of (2), and utilize this graph as an intermediate step in obtaining the final graph.

Once the graphs of (1) and (2) are obtained over the same interval of the parameter q, it is possible to obtain trial dielectric constant values directly from measured values. It is possible to see, for a single measured value, all possible values of dielectric constant within the range plotted. These two relationships could be used in the form of separate graphs on the same grid, but to facilitate evaluation, they have been combined into a single graph expressing the relationship between the measured quantity, x_0, and the unknown, K_e.

Discussion of Graph of x_0 vs K_e

Fig. 2 is an illustrative graph of the relationship: x_0 vs K_e for four values of sample length. For practical reasons, these values are integral multiples of the length $\lambda_c/4$. The value of the parameter p, the square of the ratio of wavelength to cutoff wavelength, was assumed to be 0.400.

It might be noted that curves have common points where $x_0 = 0$ and K_e equals 2.0 and 4.0. No particular significance should be attached to this feature, since the existence of common points arises from the mathematics, in that all sample lengths are multiples of the same unit; the fact that these points have even integral values is due to the value of p chosen. It is worth noting that in the interval of one cycle for the unit sample length, the order of ambiguity present for any integral multiple length is equal to the multiple itself.

Because of the mathematically indirect relationship between curves for different sample lengths, each should be considered individually. They have been plotted together to facilitate the determination of dielectric constant when more than one value of sample length is used. Different weights of line are used, one for each sample length. This set of curves will *not* permit interpolation for values of d intermediate to those for which curves are plotted. A discontinuity is present between any two curves, even between those which intersect at

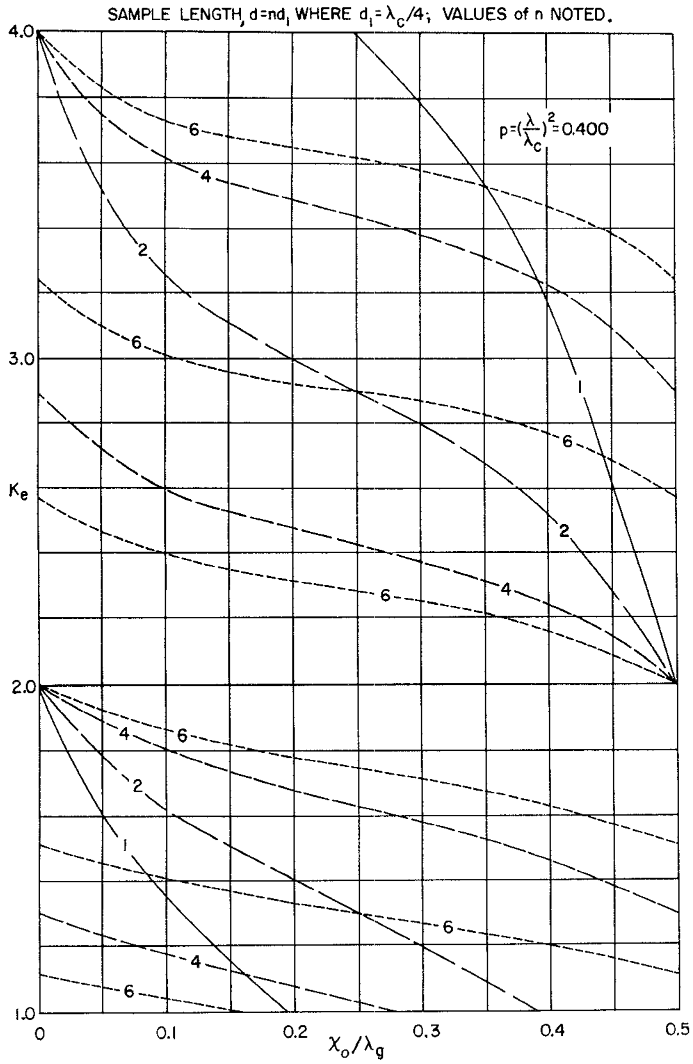

Fig. 2—Graph of dielectric constant vs measured quantity for four arbitrarily chosen sample lengths.

$x_0=0$ and thus give the illusion of being contained in the same mathematical surface. For this reason samples should be cut exactly to a length for which curves are available.

Practical Application of Fig. 2

While the method of evaluation places no inherent restriction on sample dimensions or on frequency, certain relationships must be used in order that the curves of Fig. 2 be valid. It is first necessary to choose the source frequency and waveguide dimensions so that the parameter p is equal to 0.400. This parameter, which is the square of the ratio of free-space wavelength to cutoff wavelength, sets the value of frequency for use with a given size of waveguide. If this value of p cannot be obtained with available equipment, or if for any other reason another value is desired, a modification of Fig. 2 is required.

An additional restriction is that the curves of Fig. 2 have been plotted only for sample lengths equal to integral multiples of $\lambda_c/4$, which in turn equals one-half the wide dimension of the waveguide. Other values of sample length relative to waveguide width would require further computation. However, this involves (1) only, since the length d does not enter into (2).

The above relationship of sample length to waveguide size can be used advantageously by making the waveguide inside dimensions in the ratio of exactly 2 to 1. Samples can then be cut as cubes whose edge is equal to the narrow dimension of the waveguide cavity. Two such cubes, placed side by side in the guide, form a sample corresponding to curve 1 of Fig. 2; additional samples are added to utilize the other curves. The use of cubical samples permits investigation of sample uniformity, since each cube can be inserted in any one of several orientations.

A method utilizing an adjustably mounted dial indicator circumvents several steps in the processing of data by allowing x_0 values to be read directly. This consists in merely presetting the dial indicator coupled between the probe assembly and the carriage frame to read the minimum positive value of $(N\lambda_g/2-d)$, N a positive integer, with the probe assembly resting on a voltage minimum and the sample removed. With this input, the dial indicator will read x_0 directly when, with a sample of length d in place, the probe assembly has been moved to the nearest voltage-minimum position. The range of both the dial indicator itself and that of its mounting adjustment need to be greater than $\lambda_g/2$. With this provision it is necessary only to tabulate values of dial input setting, one value for each sample length to be used.

Loss-Tangent Measurement

For evaluation of loss tangent of samples by the shorted-line method, Dakin and Works[2] give the following general expression, pertaining to the sample medium.

$$\operatorname{Tan}\delta_2=\frac{\Delta x}{d}\left[\frac{(1/\lambda_c^2+1/\lambda_g^2)-1/\lambda_c^2K_e}{1/\lambda_c^2+1/\lambda_g^2}\right]\times\frac{\beta_2 d[1+\tan^2(2\pi x_0/\lambda_g)]}{\beta^2 d(1+\tan^2\beta_2 d)-\tan\beta_2 d},\qquad(3)$$

where Δx equals the width of the standing-wave *minimum* at twice the minimum-power value (3 db above minimum indicated on swr square-law-calibrated meter), d the length of the sample, and K_e the dielectric constant of the material.

By making the substitutions: $\beta_2=2\pi/\lambda_{gs}$ and $p=(\lambda/\lambda_c)^2$, this expression is reduced to the formula:

$$\tan\delta_2=\frac{\Delta x}{d}(1-p/K_e)\qquad(4)$$

for the case where $d=n\lambda_{gs}/2$, n being an integer, and λ_{gs} the wavelength in the sample in the waveguide. With these values of sample length, $\Delta x/d$ has its minimum value and the remaining factors in the general expression (3) are insensitive to sample length.

Sample lengths for which (4) holds can be computed by use of the expression:

$$\lambda_{gs} = \lambda/\sqrt{K_e - p}. \tag{5}$$

Allowed values of sample length can be expressed directly in terms of the same measurable parameters as

$$d = \frac{n\lambda}{2\sqrt{K_e - p}}, \; n \text{ being any integer.} \tag{6}$$

Wall loss, which is a function of waveguide parameters only, is obtainable by applying (4) to the empty guide in question, for which case $K_e = 1$ and d becomes the distance from the short to the null position. Wall loss is eliminated by subtracting the value of tan δ_2 thus obtained from the value of tan δ_2 obtained with the sample in place.

A graph, from (4), facilitating evaluation of loss tangent from the value of $\Delta x/d$ measured for a sample of specified length and known K_e value is included as Fig. 3. Interpolation on this graph will permit evaluation for any practical value of the parameter p. Where wall loss needs to be calculated, it can be read also from the same graph, using the single additional necessary measurement.

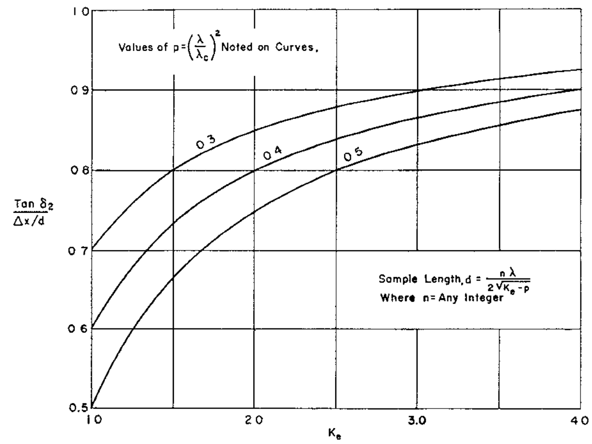

Fig. 3—Graph of multiplying factor for converting $\Delta x/d$ to loss tangent, tan δ_2.

Propagation in Ferrite-Filled Transversely Magnetized Waveguide*

P. H. VARTANIAN† AND E. T. JAYNES‡

Summary—**A solution to the problem of propagation of higher modes in a transversely magnetized ferrite-filled rectangular waveguide has been found. The solutions to the problem are expressed in the form of four rigorous nonlinear algebraic equations which characterize the problem and are ready for numerical solution. The dependence of the fields in the direction of magnetization is the same as for the classical modes.**

WE SHALL consider the problem of propagation in a rectangular waveguide which is completely filled with ferrite and magnetized transversely to the direction of propagation.

This problem is becoming of more interest as lower loss ferrites are developed. As these very low-loss ferrites become available, a class of devices depending on the ability of a dc magnetic field to change the propagation constant within a waveguide will become practical. With the transverse field geometry, these devices will operate at low field values far from gyromagnetic resonance. They will be characterized by transverse fields which are distorted by the applied magnetic field. This will make this geometry useful in field displacement devices such as isolators and radiators.

We shall hence find the fields and propagation constants for the modes in this particular ferrite geometry. They will be characterized by parameters which continuously vary with increasing magnetic field from the classical TE and TM modes into a new set of modes having fields and propagation constants which are magnetically controllable.

Gamo[1] and Kales[2] have investigated the case of the longitudinally magnetized filled cylindrical waveguide. Van Trier[3] has solved the case of the TE_{10} mode in the transversely magnetized waveguide and found that the new mode is a TE mode with a distorted transverse field dependence. Mikaelyan[4] and recently Chevalier,

* This paper was presented orally at URSI Symposium on Electromagnetic Wave Theory, University of Michigan, Ann Arbor, Mich., June 22, 1955. The work was done at the Electronic Defense Lab., of Sylvania Electric Products, Inc., under Signal Corps Contract No. DA-36-039-sc-31435, and at Stanford University.
† Electronic Defense Lab., Mountain View, Calif.
‡ Stanford Univ., Stanford, Calif.

[1] H. Gamo, "The Faraday rotation of waves in a circular waveguide," *J. Phys. Soc. Jap.*, vol. 8, p. 176; March, 1953.
[2] M. L. Kales, "Modes in waveguides containing ferrites," *J. Appl. Phys.* vol, 24, p. 609, May, 1953.
[3] A. A. Van Trier, Th. M., paper presented orally at meeting of Amer. Phys. Soc., Washington, D.C.; April, 1952.
[4] A. L. Mikaelyan, "Electromagnetic waves in a rectangular waveguide filled with a magnetized ferrite," *Doklady*, A.N. USSR, vol. 98, p. 941; October, 1954.